MATH-5C Quiz 6 14.3, 14.5

(1) Given $f(x,y,z)=x^2y^3z^4+ye^{yz}-\sqrt{4-x^2}$, find the first order partial derivatives $f_x, f_y\ f_z$.
(9 points)

$f_x(x,y,z)=2xy^3z^4+\frac{x}{\sqrt{4-x^2}}$

$f_y(x,y,z)=3x^2y^2z^4+e^{yz}+yze^{yz}$

$f_z(x,y,z)=4x^2y^3z^3+y^2e^{yz}$

Partials ⇒ use $\frac{\partial}{\partial x}$ not $\frac{d}{dx}$ or $f'(x)$

Note: Monomials are written with # first, then alphabetical

Make sure your explanation shows you really do understand what it means

(2) The table below represents $R(\theta,v)$, the range, in feet, that a ball travels if thrown with an initial speed of υ ft/sec at an angle θ degrees. (4 points)

ANGLE θ (degrees) \ SPEED v (ft/s)	75	80	85	90
35	165	188	212	238
40	173	197	222	249
45	176	200	226	253
50	173	197	222	249

a) Find $R(45,85)$ Give units and physical meaning. $R(45,85)=226$ ft. This is the range, in feet, that a ball travels if thrown with an initial speed of 85ft/m at 45°.

b) Estimate $\frac{\partial R}{\partial \theta}(45,85)$. Only one estimate needed (no need to average two). Interpret the physical meaning. Give proper units. Show work.

$\frac{\partial R}{\partial \theta}(45,85) \approx \frac{226-222}{45-40}=\frac{4}{5}$ ft/degree

OR

$\frac{222-226}{50-45}=-\frac{4}{5}$ ft/degree

$\frac{\partial R}{\partial \theta}(45,85)$ gives the instantaneous rate of change of the range of the ball relative to a change in angle while holding velocity constant

(So if we keep the initial velocity the same, the range will change approximately $\frac{4}{5}$ ft for a change of one degree)

(3) For the function f(x,y) = $\frac{x^3 y}{3x^6 + y^2}$ (SHOW WORK) (3 points each)

(a) Find $\lim_{(x,y)\to(0,0)} f(x,y)$ along any straight line y = mx. 0

$$\lim_{x\to 0} \frac{x^3 \cdot mx}{3x^6 + (mx)^2} = \lim_{x\to 0} \frac{mx^4}{3x^6 + m^2x^2} = \lim_{x\to 0} \frac{mx^2}{3x^4 + m^2}$$

(b) Find $\lim_{(x,y)\to(0,0)} f(x,y)$ along the curve y = x^3. $\frac{1}{4}$

$$\lim_{x\to 0} \frac{x^3 \cdot x^3}{3x^6 + (x^3)^2} = \lim_{x\to 0} \frac{x^6}{4x^6} = \frac{1}{4}$$

(c) What can be said about $\lim_{(x,y)\to(0,0)} f(x,y)$? It does not exis

Note: If they HAD been the same value, the limit might exist, but we cannot know for sure by checking paths.